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by

Susan M. Damms

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MOBILITY MEASUREMENTS ON A BEAM AND A DYNAMIC ABSORBER

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10 Susan M. Damms

SUMMARY

This Memorandum describes the measurement of the responses of two structures to certain input forces. The main object of the experiment was to use the measurements performed on the two separate structures to predict the results for a structure formed by connecting them together. One of the structures chosen was a beam and the other was designed to behave as a dynamic absorber when attached to the beam. Confidence was given to the measurements by comparing the results with theory as far as possible.

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1 INTRODUCTION

This Memorandum describes the measurement of mobilities on each of two single structures and then on a structure formed by connecting the two together. Mobility is measured by determining the responses of a structure to a set of applied input forces and/or moments. Here it will be assumed that the systems involved are linear so that the response to a force with any time history can be obtained from the results of harmonic excitation through Fourier transformation. Also, the information required to describe the vibrational properties of a structure which responds linearly involves only the ratio of response to input and not the two quantities separately. The measurement and use of mobilities is dealt with extensively in the literature. Ref 1 provides an introduction to the subject.

The two structures chosen were a beam and a structure which would behave as a dynamic absorber when attached to the beam. The frequency range covered was 20-400 Hz which included the first four modes of the beam. The main object of the experiment was to check that mobility measurements on the two separately could be used to predict those for the connected structure. The theory behind this is well-established and is given here in section 2 followed by a description of how a dynamic absorber works. Refs 2 and 3 provide examples of applications of the theory. The test pieces were selected so that the experimental results could be checked by theory as far as possible. Another factor in the choice was that the only significant reaction between them when connected should be the normal force of reaction. The only mobility measurements made were for force inputs parallel to this direction. In general, the mobility data for the separate components required to predict those for an assembled structure are those corresponding to any significant force and moment of reaction between the components.

An underlying aim of the present work was to obtain experience in mobility measurements. The next stage is to include mobilities which involve moment inputs and rotational responses. Also, the frequency range used in the current experiment was chosen to include only the first four modes of the beam. It is intended to extend the work to cover systems with high modal density. The end objective is to design an interface to isolate vibrationally the gearbox of a helicopter from its cabin to achieve a reduction in the internal noise. This is expected to involve the measurement of the cabin and gearbox mobilities at their respective points of attachment. The mobilities of the proposed interface can

then be used to calculate the behaviour of the assembled structure and the interface optimised to obtain the best overall characteristics.

The layout of the Memorandum is as follows. Section 2 provides the theory for the derivation of the mobilities of an assembled structure from those of its components. A description of how a dynamic absorber works utilises this theory and is given in the same section. Other theory required is more specific to the test pieces chosen and is given in the Appendix. Section 3 provides a description of the experimental procedure and the results are discussed in section 4.

2 THEORY

The theory for the work described in this Memorandum falls into two categories: that required to calculate the frequencies and responses of the particular structural test pieces chosen when subject to free or moment excitation and that which predicts the responses of a total assembly when measurements on the individual components of the assembly are known. The theory for the latter will be described in this section since it is central to work on the longer term objective of vibration isolation; it also leads to a description of how a dynamic absorber works. Any other theory required in this paper will be given in the Appendix since it is of a more specific nature.

At any particular point of a given structure and at a given frequency there are six possible independent vibrational inputs, namely a force in any of three orthogonal directions or a moment about any of three orthogonal directions. The response to any one of these inputs at each point of the structure has six independent components, namely translation in and rotation about each of three orthogonal directions.

Suppose that a structure is excited at one or more points by m independent inputs (p_1, \dots, p_m) some of which will be forces and some moments. Because of linearity the corresponding m responses (x_1, x_2, \dots, x_m) are related to the forces by the matrix equation

$$\underline{X} = [Y] \underline{P} \quad (1)$$

where Y is a frequency dependent $m \times m$ matrix with complex entries, X is the column vector (x_1, x_2, \dots, x_m) and P is the column vector (p_1, p_2, \dots, p_m) . The inputs are independent if Y is nonsingular. By corresponding response to an input is meant the translation in the direction of a force input at its point of application or the rotation about the direction of a moment input at its

point of application. The (i,j) th element y_{ij} of Y is a transfer function and is a measure of the response x_i to the input p_j so that it has dimensions of response/input. When $i = j$ this is a direct transfer function and is a cross transfer function otherwise. The transfer functions will be referred to as mobilities in general with a specific name given to the matrix Y to distinguish which of displacement, velocity or acceleration is used as the measure of response. These quantities are of course simply related for a given frequency. In the current work acceleration was used and Y will be referred to as the inertance matrix.

The inverse of Y plays an important role in what follows. It is often called the 'acceleration impedance' matrix and here will just be called the impedance matrix. If $Z = Y^{-1}$ then

$$\underline{P} = [Z]\underline{X} \quad (2)$$

The (i,j) th element of Z , z_{ij} , is the value of the i th input (force or moment) when $x_j = 1$ and $x_k = 0$, $k \neq j$. The element z_{ij} depends on all the elements of Y and care must be taken not to confuse it with y_{ij}^{-1} . This is also referred to as an impedance but is the value of the i th input when $x_j = 1$ and all the other externally imposed forces and moments are equal to zero.

The simplest example of an assembled structure is one formed of two components where there is a force input at the single point junction and where the reaction between the two is a force with the same direction as the externally applied force. The situation is illustrated in Fig 1. The excitation and response for each component at the points where they are to be connected are given by:

$$p_a = z_a x_a$$

and

$$p_b = z_b x_b$$

When A and B are connected to form C then

$$x_a = x_b = x_c$$

and

$$p_a + p_b = p_c$$

so that

$$p_c = (z_a + z_b)x_c$$

Hence

$$z_c = z_a + z_b . \quad (3)$$

The argument extends to the situation where the two components are to be joined together at m junctions where a junction is defined as only being able to transmit a force or moment in one direction. A connection at a single point may thus comprise up to six junctions. The above equations then become matrix equations with the conclusion

$$Z_C = Z_A + Z_B . \quad (4)$$

More generally there will be inputs and responses of interest other than those at the junctions of A and B . Suppose then that there are k junctions between A and B . Let the input and response column matrix elements for A be ordered so that the last k elements correspond to these junctions and those for B be ordered so that the first k elements correspond to the junctions. Then the impedance matrices Z_A and Z_B can be partitioned as follows:

$$[Z_A] = \begin{pmatrix} z_A^{11} & z_A^{12} \\ z_A^{21} & z_A^{22} \end{pmatrix}$$

and

$$[Z_B] = \begin{pmatrix} z_B^{11} & z_B^{12} \\ z_B^{21} & z_B^{22} \end{pmatrix}$$

where z_A^{22} and z_B^{11} are both $k \times k$ matrices and are the elements of Z_A and Z_B respectively corresponding to the junctions between them. Then the resulting impedance matrix for C is given by

$$[Z_C] = \begin{pmatrix} z_A^{11} & z_A^{12} & 0 \\ z_A^{21} & z_A^{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & z_B^{11} & z_B^{12} \\ 0 & z_B^{21} & z_B^{22} \end{pmatrix} . \quad (5)$$

The size of Z_C is correct since it relates to the total number of inputs and responses of interest in the assembled structure.

The above theory plays a role in the explanation of how a dynamic absorber works. A dynamic absorber is a relatively lightweight device tuned to a given frequency which, when attached to a structure, reduces the level of vibration at that frequency. An essential feature of the absorber is that its impedance is high (at the relevant frequency) compared with the structure at their respective points of attachment to each other. The simplest example of a dynamic absorber is a spring attached to a mass in series. The spring plus mass has a resonant frequency of $\sqrt{k/m}$ when the free end of the spring is grounded where k is the spring stiffness and m is the value of the mass. Now the resonant frequency of this grounded system is precisely the antiresonant frequency of the system which would result if the grounded end of the spring was to be uprooted and used as the drive point⁴. It can be shown that the magnitude of the impedance at this drive point is m/η where η is the damping of the absorber as a fraction of the critical damping. Let the situation be illustrated by Fig 1 where the structure is denoted by A, the absorber which is attached by the free end of the spring by B and the composite by C. Then equation (3) applies and it is clear that if $|z_b|$, which equals m/η at the absorber frequency, is much greater than $|z_a|$ then the presence of the absorber will reduce the level of vibration significantly at the attachment point. If the frequency is a resonant frequency of the structure and the absorber is attached at a point where the normal mode response dominates then vibrations must be similarly reduced at those other points of the structure where the normal mode response dominates. This is because the mode shape gives the relative levels of the motion at these points. In general, to discover whether vibrations are reduced on the structure at points other than that of the attachment of the absorber and also whether the absorber is effective when it is not attached at the position of external excitation the results of applying equation (5) must be considered. The word 'absorber' is something of a misnomer since the principle is not one of energy absorption but purely of reducing the response at the frequency of interest. Since the natural frequencies are changed the response at some frequencies will be increased.

3 EXPERIMENTAL DETAILS

Throughout the experiment the excitation input was a force in one direction only. No moments were externally applied. The responses measured were accelerations in the same direction and limited white noise provided by a random noise generator was used for excitation. Accelerometer and force gauge readings were fed through charge amplifiers to a Fourier analyser which used digital fast

Fourier transforms. The frequency range used for analysis was 0-400 Hz with an analysis bandwidth of 0.78 Hz. The sample length was 1.28 seconds and fifty overlapping samples were taken.

The work consisted of three stages after performing the necessary calibrations. Firstly, a thin free-free beam with slight non-uniformities described below was forced in a direction normal to its plane at each of two points. For each position of forcing the response was measured at both points using accelerometers aligned with the direction of forcing. These measurements were then used to derive direct and cross inertances. The second stage of the experiment was to take measurements on a small structure which would behave as a dynamic absorber when attached to the beam. This absorber was tuned to a natural frequency of the beam. The third stage of the experiment was to fix the absorber to the beam at one of the excitation positions and repeat the set of measurements made in the first stage. One of the objectives of the experiment was to predict the results of this final stage from those of the first two.

The beam used in the first stage of the experiment is illustrated in Fig 2 and was a uniform aluminium beam 0.99 m long, 0.037 m wide and 0.0064 m deep with attachments. These attachments were considered to be part of the basic structure and comprised damping material and two accelerometers as described below. The beam was supported with its plane surface horizontal by means of four lengths of cotton attached to a supporting frame by valve rubber. The cotton was tied to the beam through small holes 0.13 m from its two ends and near each of the two long edges. This support had a very low natural frequency so that the beam behaved effectively as a free-free beam over the frequency range of interest. A free-free beam was chosen rather than a clamped-free beam since there is difficulty in isolating the clamp from its surroundings. Three strips of viscoelastic damping material 3 mm thick, 0.12 m long and with the same width as the beam were glued to the beam. Without these added the damping of the beam would have been a fraction of one per cent which is much lower than values of general practical interest. When the damping is so low the input force goes through an extremely small and sharp minimum at resonance and very narrow bandwidths must be used for analysis in the neighbourhood of the resonance. Two 30 gram accelerometers were screwed to the beam on its centre line at positions 0.30 m and 0.64 m from one end. These were the locations chosen for attachment of the exciter and for response measurement. Although an impedance head was used to measure force input and this comprises an accelerometer and a force gauge housed in the same casing, the two added accelerometers were necessary for cross transfer

measurements. Rather than having to compensate for their masses in the response measurement it was deemed better to treat them as part of the basic structure.

The impedance head was fixed to the beam at the desired position of excitation and the exciter was connected to the head by means of a rod aligned with the direction of excitation. This rod was about 80 mm long with a 15 mm section of piano wire which was included because its lateral flexibility reduces the transmission of moments and lateral forces. Previously a longer connecting rod had been used but transverse vibrations in the rod caused apparent extra resonances in the transfer function.

Response measurements were made with both accelerometers for forcing at each of the two excitation points in turn. The two direct and the two cross inertances for the points were then obtained from the results.

The next step was to perform those measurements on the dynamic absorber that were necessary, in conjunction with the data already acquired for the beam, to predict the behaviour of the beam with the absorber attached. The absorber is illustrated in Fig 3 and consisted of a steel strip 16 mm wide and 0.91 mm thick with two similar masses of 35g at its ends. At the midpoint of the strip there was a hole through which a stud was passed to provide fixing to the beam. Two 16mm diameter washers threaded on to the stud on either side of the strip were to be held firmly in position to provide a form of clamp. A nut threaded on the stud was also considered to be part of the absorber. The absorber thus consisted essentially of a pair of similar cantilever beams with masses at their free ends. To make the measurement the stud of the absorber was threaded into the impedance head which was attached directly to the exciter. The outputs from the force gauge and the impedance head accelerometer were analysed to provide the direct inertance of the absorber at the point of excitation.

In the final part of the experiment the absorber was attached at the accelerometer position on the beam 0.30 m from its end. The absorber was mounted so that its 'arms' were transverse to the beam as illustrated in Fig 4. This alignment was chosen to minimise any moment of reaction between the beam and the absorber which would be in the right direction to excite bending waves in the beam. The intention was that the only important reaction between the two would be the normal force. Otherwise inertance measurements for moment and other force directions would have been necessary in the first two stages of the experiment to be able to predict the outcome of the final stage. The same

measurements were carried out as for the beam on its own so that two direct and two cross inertances were obtained.

4 RESULTS

All the figures referred to in this section show the magnitude of inertance as a function of frequency over the range 30 Hz to 350 Hz. The excitation and response points at 0.30 m and 0.64 m from the end of the beam will be referred to as points 1 and 2 respectively. The damping was typically of the order of 2% and the analysis bandwidth was 0.78 Hz so that the magnitude of the resonance peaks and the antiresonance troughs may not be accurately represented. Corrections have been applied for the mass below the force gauge.

The solid lines in Figs 5 and 6 show the respective magnitudes of the direct inertances for the points 1 and 2 on the beam without the absorber attached. By comparing Fig 5 with Fig 6 it can be seen that point 1 is near a node of the third mode (frequency = 181 Hz). The dashed lines in the same two figures are theoretical results corresponding to the two measurements. The theory used is given in the Appendix and includes the mass and stiffness of the added damping material. The damping itself was taken to be zero since this was only expected to affect the sharpness of the resonance peaks and the anti-resonance troughs. In the calculations it was necessary to assume a value for the Young's modulus of the damping material. The value used corresponded to that provided in the manufacturer's data for testing at 75 Hz at a temperature of 15°C. The experiment was actually performed at 20°C at which temperature the Young's modulus at 75 Hz was given to be 30% lower. This corresponded to a 5% lower bending stiffness of the cross-section and gave less good agreement with experiment.

The solid line and the dashed line in Fig 7 show the two measured cross inertances for the beam without the absorber, *ie* the ratio of the response at 2 to the force at 1 and the ratio of the response at 1 to the force at 2. The measurements agree with reciprocity, namely that the ratio of the exciting force to the observed acceleration remains the same if the excitation and observation points are interchanged, provided also that the direction in which the force acts in each case is the same as that in which the acceleration is measured in the other case.

Fig 8 shows the direct transfer function for the absorber which had been tuned to have its highest impedance very close to 89 Hz which was the second natural frequency of the beam. The frequency was checked by a calculation

performed on the following basis. The antiresonant frequency is the resonant frequency of the same system when it has been grounded at the point at which it was previously forced. Now resonant frequencies are independent of the drive point and the grounded absorber is effectively a pair of cantilever beams clamped at their common root and with masses added at their ends. One half of the natural frequencies will result from the symmetric motion of the 'arms' and hence be identical to the natural frequencies of one of the cantilevers plus added mass. The theory for adding masses to a non-damped system with known mode shapes is included in the Appendix, and this was used to find the natural frequencies by calculating the frequencies with zero impedance. It was found to be important to include the moment inertia of the added masses which meant taking the moment impedance of the cantilever at the point of attachment into account.

The absorber was attached to the beam at point 1. The solid line in Fig 9 shows the point inertance at point 1 and for comparison the dashed line shows the same without the absorber present. As intended the absorber greatly reduced the inertance at its tuned frequency to the extent of introducing an anti-resonance there. However this was at the expense of increases elsewhere since the effect of the absorber was to shift the existing natural frequencies and introduce another.

From the theory in section 2 it is possible to predict the transfer functions for the beam plus absorber from those for the beam and absorber separately. Let Y_{11} and Y_{22} denote the direct inertances for the beam alone at the points 1 and 2 respectively and Y_{12} denote the cross inertance for forcing at point 2 and acceleration measurement at point 1. Because of reciprocity the cross transfer function Y_{21} with forcing at point 1 and acceleration measurement at point 2 equals Y_{12} . Let Y_a denote the inertance of the absorber and denote by Y'_{11} , Y'_{22} , Y'_{12} the inertances corresponding to Y_{11} , Y_{22} and Y_{12} after the absorber has been fixed to the beam. Then, providing the only significant reaction between the beam and the absorber is the normal force, the theory predicts

$$Y'_{11} = \left(Y_{11}^{-1} + Y_a^{-1} \right)^{-1} \quad (6)$$

$$Y'_{22} = Y_{22} - Y_{12}^2 (Y_a + Y_{11})^{-1} \quad (7)$$

and

$$Y'_{12} = Y_{12} Y_a (Y_a + Y_{11})^{-1} \quad (8)$$

The direct inertance at point 1 predicted from the separate measurements on the beam and the absorber using equation (6) is shown by the solid line in Fig 10. This agrees well with the measured transfer function which is represented by the dashed line.

The solid line in Fig 11 shows the direct inertance at point 2, that is at a point where the absorber is not attached. Unlike the direct inertance at point 1, there is no antiresonance at the frequency to which the absorber is tuned although the inertance has still been substantially reduced from that for the beam alone. The dashed line shows the prediction from the separate measurements on the beam and the absorber using equation (7). The agreement in the neighbourhood of the absorber frequency is not as good as elsewhere. Reference to equation (7) gives an indication of the probable cause of the problem. The values of Y_{11} , Y_{22} and Y_{12} are all large since the beam was resonant very close to that frequency and Y_a is small. The operations on the right hand side of (7) involve differencing large quantities to provide an answer which is not large. Under these circumstances inaccuracies in the measurements around the peaks of Y_1 , Y_2 and Y_{12} , which is precisely where the inaccuracies are greatest, may well lead to discrepancies like those shown around the absorber frequency in Fig 11.

Fig 12 shows the two cross inertance measurements Y'_{12} and Y'_{21} . These should be equal and the agreement is reasonable. The solid line in Fig 13 shows the predicted inertance from equation (8) with one of the actual measurements represented by the dashed line for comparison. The agreement is good. The antiresonance is at the tuned absorber frequency.

5 CONCLUSIONS

The mobility measurements performed on the beam and the dynamic absorber separately were used to give a satisfactory prediction of the measurements on the structure formed by connecting them together. The results for the beam alone showed good agreement with theory and this gave further confidence to the quality of the measurements. Thus it has been demonstrated that for a test case involving low modal density and simple force excitation accurate mobility data can be obtained in practice.

Appendix

This Appendix describes the theory for the response of the beam used in the experiment including the effects of the added damping material and the accelerometer masses. The theory is in two stages. Firstly, the derivation of the mode shapes for the basic beam with the added damping material is described. This is done by considering a beam with discontinuities in bending stiffness and mass per unit length. The value of the damping was not taken into account since this was only expected to modify the shape of the resonance peaks and the anti-resonance troughs without any significant shift in frequency. Secondly, to find the response of the beam with the two accelerometer masses added, the moment and force transfer functions at the points of addition were calculated from the mode shapes for the basic beam plus damping material. Calculation of the response then follows from the application of the theory given in section 2 since the moment and force transfer functions for the added masses are known.

Suppose the beam, which is thin, lies along the x-axis from $x = 0$ to $x = \ell$ with the y-direction normal to its plane and the z-direction orthogonal to x and y. Denote the acceleration at x in the y-direction by $\phi(x, t)$. Then, providing no external forces or moments are present the equation of motion for bending waves is:

$$-\frac{\partial^2 \phi}{\partial t^2} = \frac{B}{m} \frac{\partial^4 \phi}{\partial x^4}$$

where m = mass per unit length, and B = bending stiffness. The beam can be split into those sections where there is no added damping and those with damping. On each section B and m are constant but there are discontinuities between sections. Suppose there are n such sections with $x = x_1, \dots, x_{n-1}$ denoting the discontinuities. Define $x_0 = 0$ and $x_n = \ell$. Then on the j th section

$$-\frac{\partial^2 \phi}{\partial t^2} = \frac{B_j}{m_j} \frac{\partial^4 \phi}{\partial x^4} \quad x_{j-1} \leq x \leq x_j.$$

Suppose single frequency excitation and write

$$\phi = \tilde{\phi}(x) e^{i\omega t}$$

so that

$$\omega^2 \tilde{\phi} = \frac{B_j}{m_j} \frac{\partial^4 \tilde{\phi}}{\partial x^4} \quad x_{j-1} \leq x \leq x_j.$$

This equation has a solution of the form

$$\tilde{\phi} = P_j \sin k_j x + Q_j \cos k_j x + R_j e^{k_j(x-x_j)} + S_j e^{k_j(x_{j-1}-x)} \quad x_{j-1} \leq x \leq x_j,$$

where $k_j^4 = \frac{m_j \omega^2}{B_j}$.

Since the beam is a free-free beam the following conditions must hold:

$$\frac{\partial^2 \tilde{\phi}(0)}{\partial x^2} = \frac{\partial^3 \tilde{\phi}(0)}{\partial x^3} = \frac{\partial^2 \tilde{\phi}(l)}{\partial x^2} = \frac{\partial^3 \tilde{\phi}(l)}{\partial x^3} = 0.$$

Also $\tilde{\phi}$ and its derivatives must be continuous at $x = x_j$, $j = 1, \dots, n-1$. Altogether this gives $4n$ conditions from which P_j, Q_j, R_j, S_j , $j=1, \dots, n$ can be determined and, since P_1 can be treated as a scaling factor and be chosen arbitrarily, these conditions also determine the eigenvalues of ω . The problem was solved iteratively on a computer using the eigenvalues for a uniform beam as starting values.

Once the mode shapes $\tilde{\phi}_1(x), \tilde{\phi}_2(x), \dots, \tilde{\phi}_N(x), \dots$ have been determined the acceleration response $a(\xi_2, t)$ at $x = \xi_2$ resulting from a point force input $F(t)$ in the y -direction or a moment input $C(t)$ about the z -direction at $x = \xi_1$ can be written down. First define the Fourier transforms of the variables a, F and C as follows:

$$\tilde{a}(\xi_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} a(\xi_2, t) e^{-i\omega t} dt,$$

$$\tilde{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t) e^{-i\omega t} dt$$

and

$$\tilde{C}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(t) e^{-i\omega t} dt .$$

Then the acceleration response to the point force is given by

$$\tilde{a}(\xi_2, \omega) = \sum_{n=1}^{\infty} \frac{\omega^2}{\omega^2 - \omega_n^2} \frac{\tilde{\phi}_n(\xi_2) \tilde{\phi}_n(\xi_1) \tilde{F}(\omega)}{\Lambda_n} \quad (A-1)$$

and the acceleration response to the point moment is given by

$$\tilde{a}(\xi_2, \omega) = \sum_{n=1}^{\infty} \frac{\omega^2}{\omega^2 - \omega_n^2} \frac{\tilde{\phi}_n(\xi_2)}{\Lambda_n} \left. \frac{\partial \tilde{\phi}_n}{\partial x} \right|_{x=\xi_1} \tilde{C}(\omega) \quad (A-2)$$

where $\tilde{\phi}_n(x)$ is the nth mode shape, ω_n the corresponding eigenfrequency and

$$\Lambda_n = \int_0^l m(x) \tilde{\phi}_n^2(x) dx . \quad (A-3)$$

In this expression $m(x)$ is the mass per unit length at x . The Fourier transform, $\tilde{\Omega}(\xi_2, \omega)$, of the angular acceleration at $x = \xi_2$ is given by

$$\tilde{\Omega}(\xi_2, \omega) = \left. \frac{\partial \tilde{a}(x, \omega)}{\partial x} \right|_{x=\xi_2} \quad (A-4)$$

so that the angular acceleration response to both a point force and a point moment can be written down.

From the expressions given above it is possible to obtain the direct transfer functions at the two points $x = \mu_1, \mu_2$ at which the two masses are added and also the cross-transfer functions between the two points. The transfer functions for moment input are included. This is because the moments of inertia of the two masses may be large enough for a significant moment as well as a significant force of reaction to be induced between the beam and the masses. When both forces in the y-direction and moments about the z-direction

are the excitation inputs to be considered, the inertance and corresponding impedance matrix which concern the two points of interest will be of size 4×4 . At each point there are two possible inputs, a force or a moment, and corresponding to each input there is an acceleration and an angular acceleration response. Let Y_{ij} denote the (i,j) th element of the inertance matrix Y where $j = 1$ corresponds to force input at $x = \mu_1$, $j = 2$ to moment input at $x = \mu_1$, $j = 3$ to force input at $x = \mu_2$, $j = 4$ to moment input at $x = \mu_2$, $i = 1$ to acceleration response at $x = \mu_1$, $i = 2$ to rotational response at $x = \mu_1$, $i = 3$ to acceleration response at $x = \mu_2$, $i = 4$ to rotational response at $x = \mu_2$ etc. Thus, for example, Y_{34} is the acceleration response at $x = \mu_2$ resulting from a unit moment input at $x = \mu_2$. The mobility matrix for each mass corresponding to the point at which it is to be fixed to the beam is of order 2×2 . If the first mass has mass m_1 and moment of inertia I_1 then its mobility matrix is

$$\begin{pmatrix} m_1^{-1} & 0 \\ 0 & I_1^{-1} \end{pmatrix}$$

and the impedance matrix is

$$\begin{pmatrix} m_1 & 0 \\ 0 & I_1 \end{pmatrix}.$$

The matrices for the second mass are similar with m_2 and I_2 replacing m_1 and I_1 .

The theory in section 2 indicates that the impedance matrix Z' for the beam with added masses is given by

$$[Z'] = [Y]^{-1} + [Z_m]$$

where $[Z_m] = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & I_2 \end{pmatrix}$ so that the mobility matrix Y' is given by

$$\begin{aligned} [Y'] &= ([Y]^{-1} + [Z_m])^{-1} \\ &= [Y] (1 + [Y][Z_m])^{-1}. \end{aligned}$$

Since Y and Z_m are known this expression can be used for evaluating the required inertances. The elements of Y are derived from equations (A-1) to (A-4) and can be seen to be frequency dependent. In order to make the calculation possible only a finite number of modes can be included so that the summation must be truncated.

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3	D.J. Ewins M.G. Sainsbury	Mobility measurements for the vibration analysis of connected structures. <i>The Shock and Vibration Bulletin</i> , <u>42</u> , Part 1, pp.105-121 (1972)
4	J.P. Salter	<i>Steady state vibration.</i> Havant, Kenneth Mason (1969)

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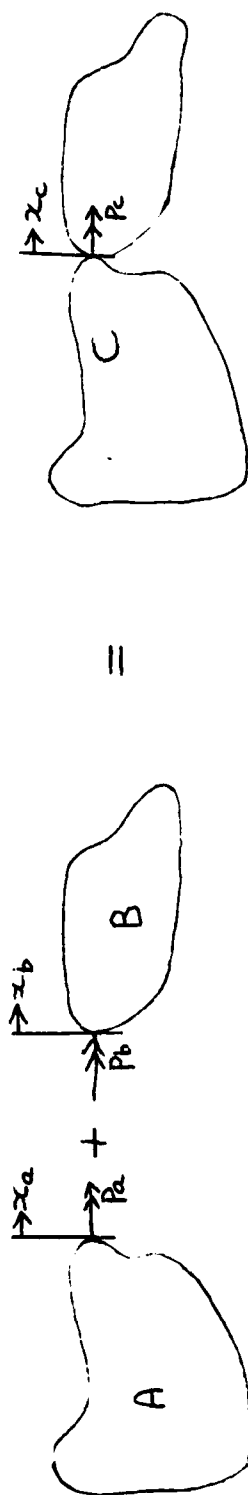


Fig 1

Fig 1 An assembled structure formed from two components

Fig 2

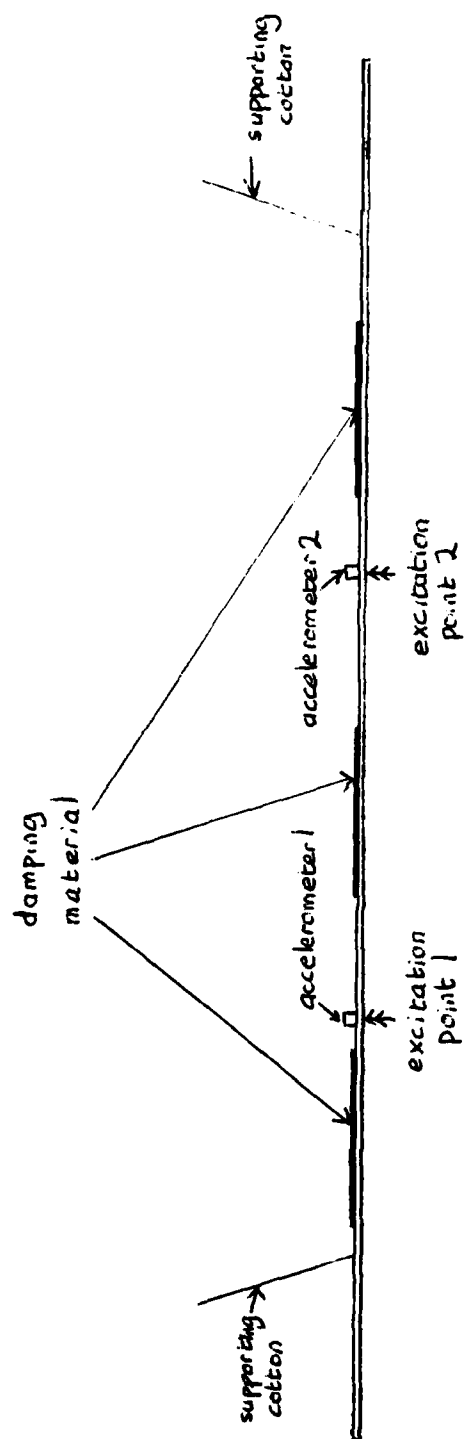
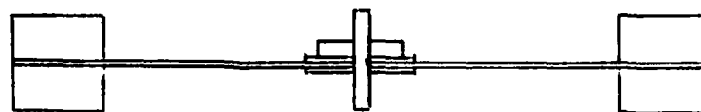
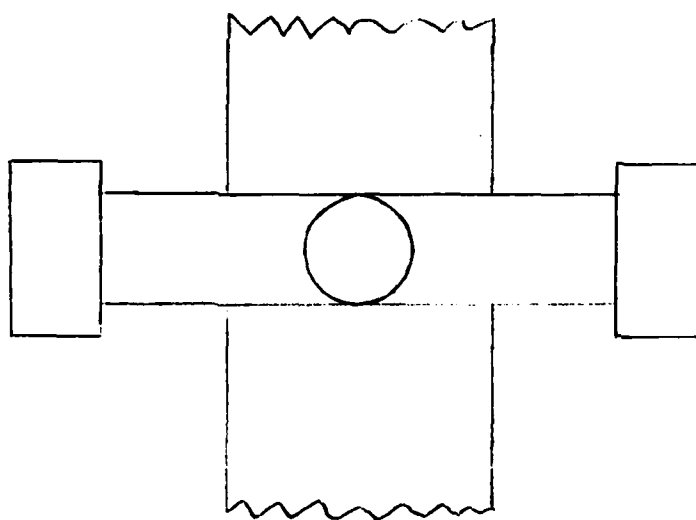


Fig 2 Beam with added damping and accelerometers



Side view

Fig 3 Absorber



Plan view

Fig 4 Absorber attached to beam

Figs 5&6

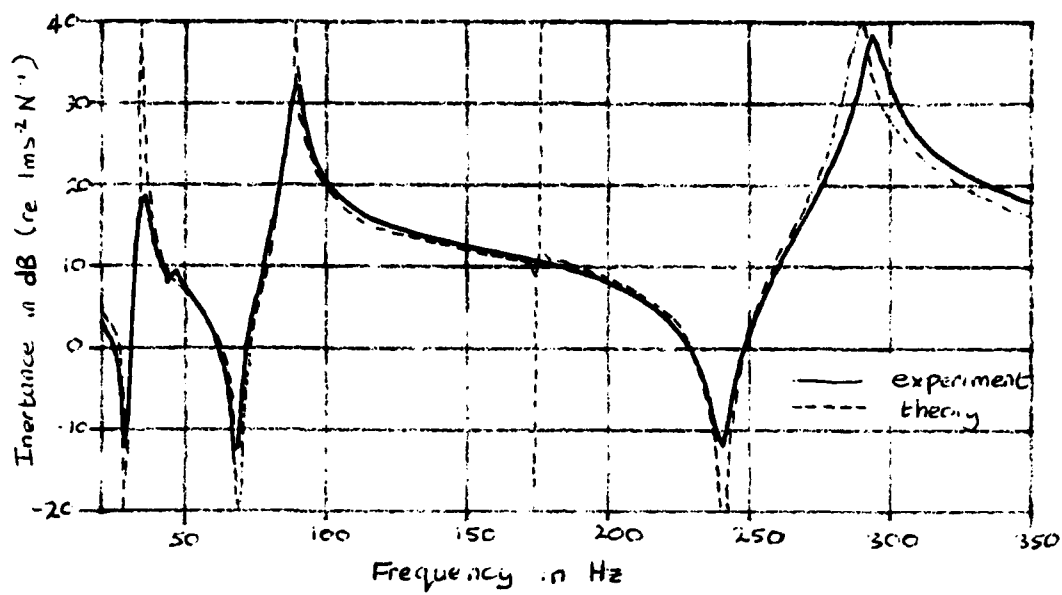


Fig 5 Direct inertance of beam at point 1

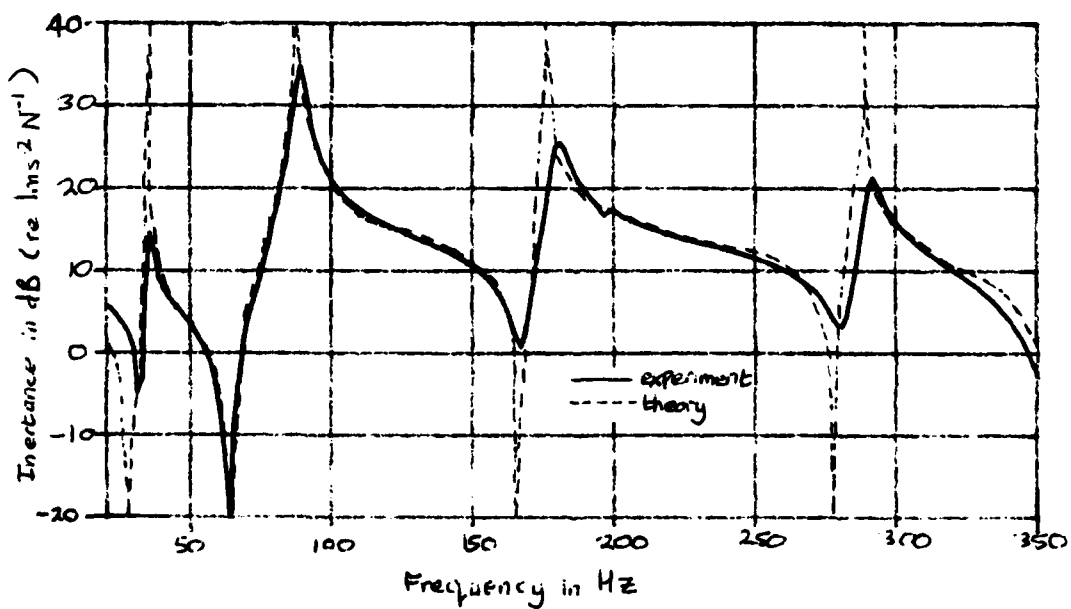


Fig 6 Direct inertance of beam at point 2

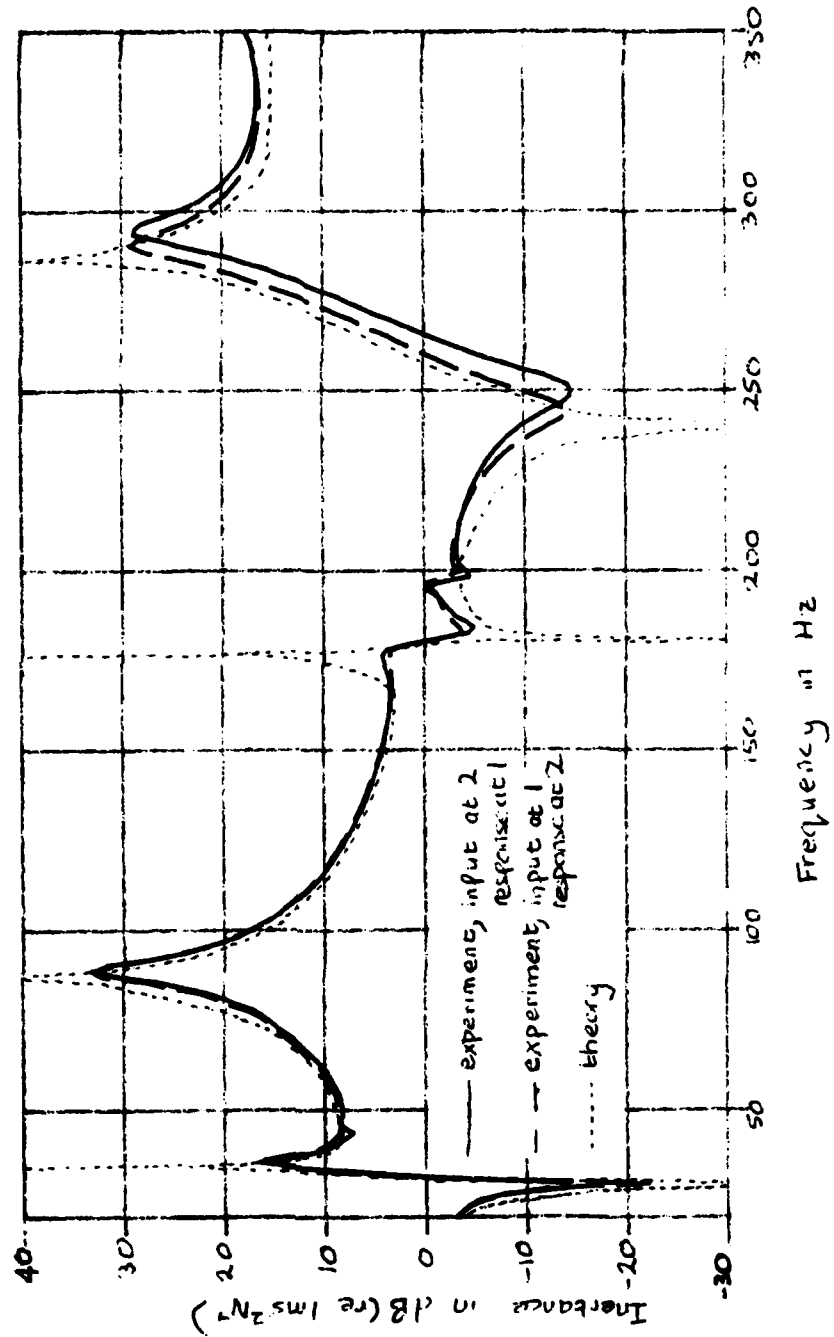


Fig 7

Fig 7 The two cross inertances between points 1 and 2 of the beam

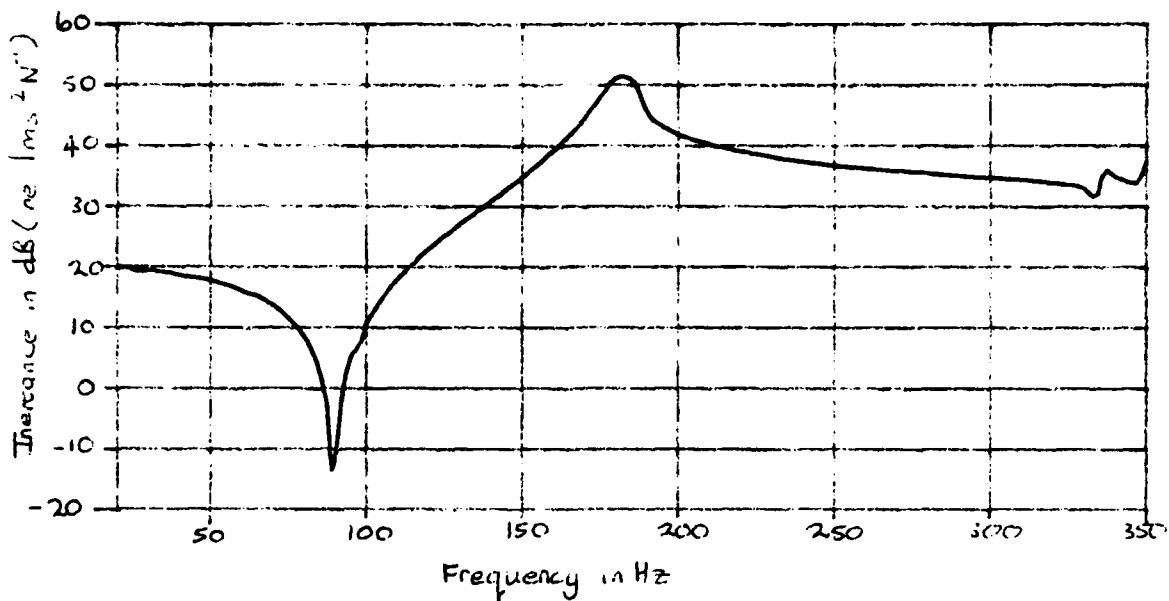


Fig 8 Inertance of the absorber

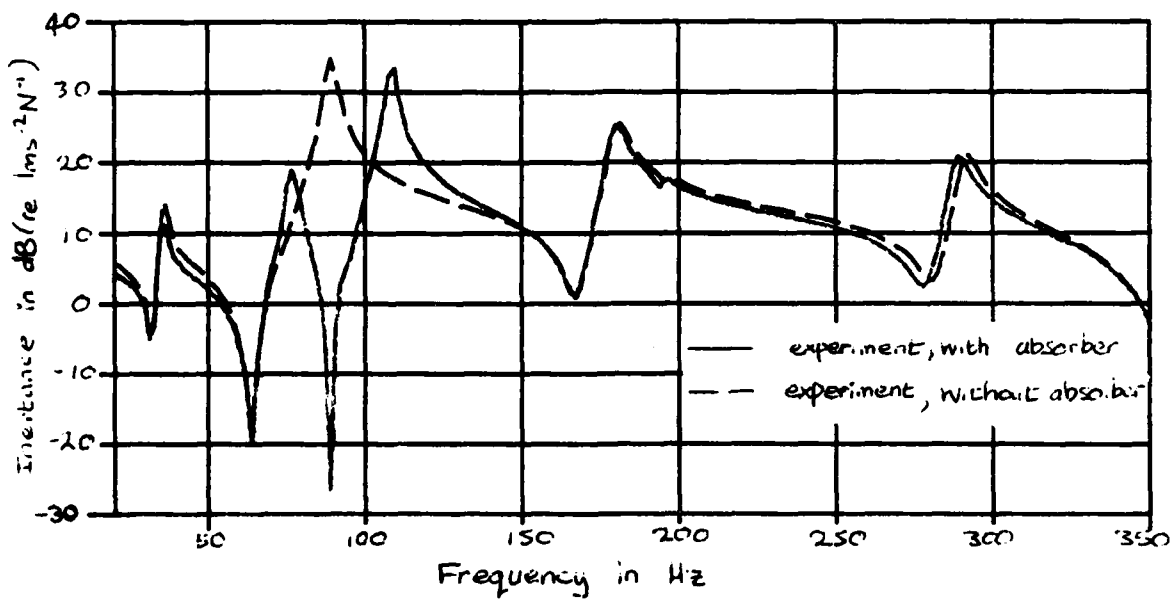


Fig 9 Direct inertance at point 1 of beam with absorber attached

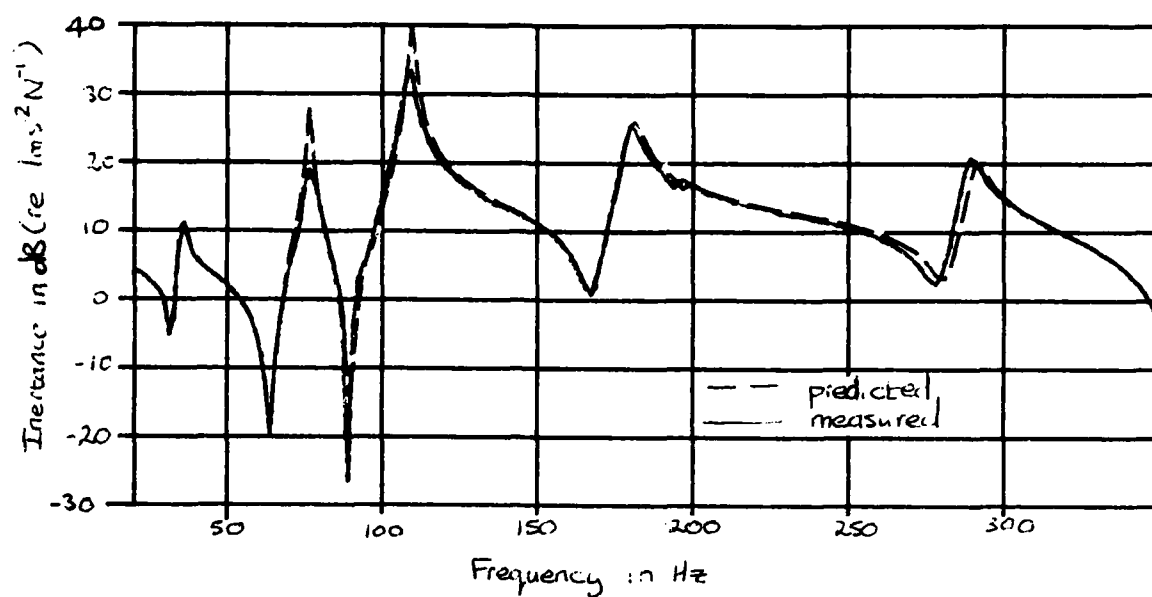


Fig 10 Predicted and measured direct inertance at point 1 with absorber

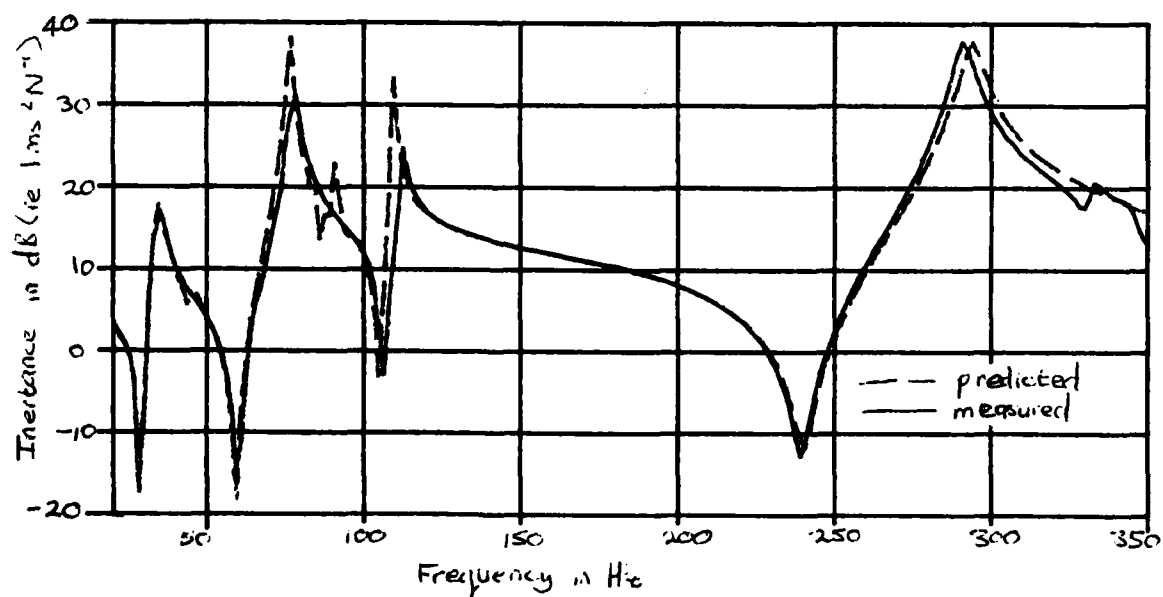


Fig 11 Predicted and measured direct inertance at point 2 with absorber

Figs 12&13

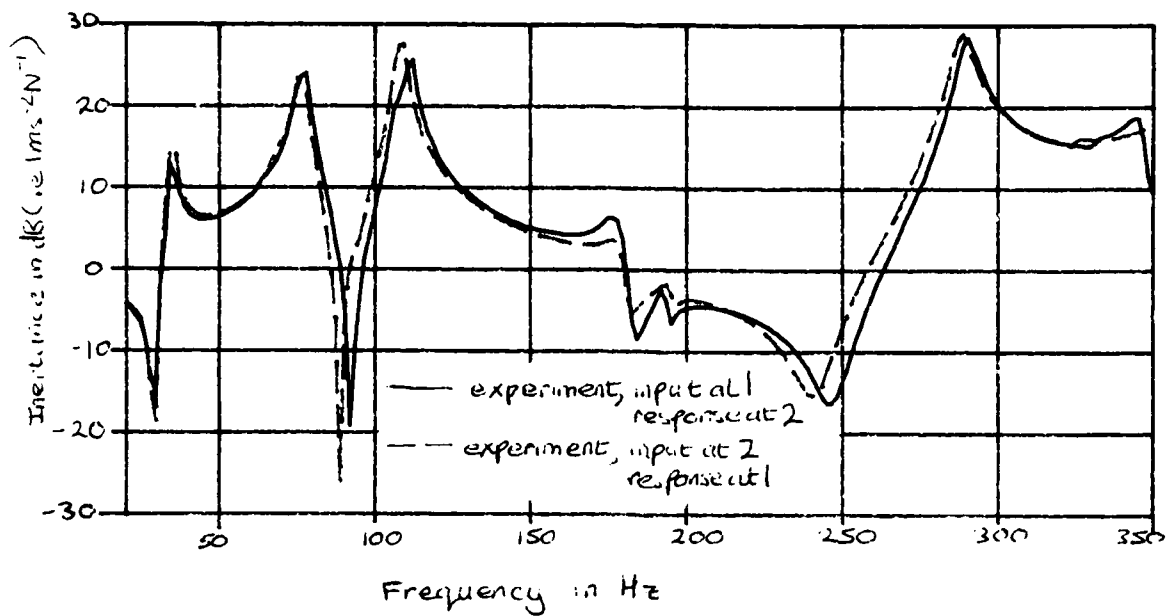


Fig 12 Measured cross inertances of the beam with absorber

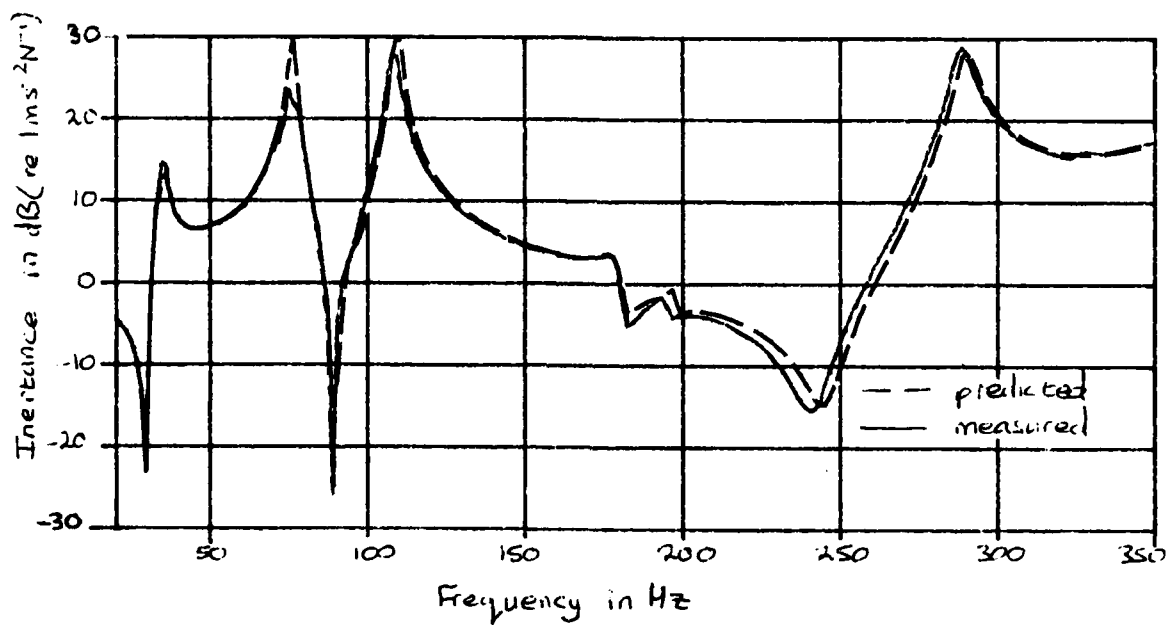


Fig 13 Comparison between predicted and measured cross inertia with absorber

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17. Abstract This Memorandum describes the measurement of the responses of two structures to certain input forces. The main object of the experiment was to use the measurements performed on the two separate structures to predict the results for a structure formed by connecting them together. One of the structures chosen was a beam and the other was designed to behave as a dynamic absorber when attached to the beam. Confidence was given to the measurements by comparing the results with theory as far as possible.					

